

# Erraticity Analysis of Soft Production by ECOMB

Zhen Cao<sup>(1)</sup> and Rudolph C. Hwa<sup>(2)</sup>

<sup>(1)</sup>High Energy Astrophysics Institute, Department of Physics,  
University of Utah, Salt Lake City, UT 84112

<sup>(2)</sup>Institute of Theoretical Science and Department of Physics  
University of Oregon, Eugene, OR 97403-5203

## Abstract

Event-to-event fluctuations of the spatial patterns of the final states of high-energy collisions, referred to as erraticity, are studied for the data generated by a soft-interaction model called ECOMB. The moments  $C_{p,q}$  do not show simple power-law dependences on the bin size. New measures of erraticity are proposed that generalize the bin-size dependence. The method should be applied not only to the soft production data of NA22 and NA27 to check the dynamical content of ECOMB, but also to other collision processes, such as  $e^+e^-$  annihilation and heavy-ion collisions.

## 1 Introduction

Progress in the study of multiparticle production has recently been made in two distinct directions among many others. One is in finding measures of event-to-event fluctuations [1] that can probe the production dynamics more deeply than the conventional observables, such as multiplicity distribution and factorial moments [2]. Such measures have been referred to as erraticity [3], which quantifies the erratic nature of the event structure. The other direction is in the construction of a Monte Carlo generator, called ECOMB [4], that simulates soft interaction in hadronic collisions, capable of reproducing the intermittency data [5]. ECOMB stands for eikonal color mutation branching, which are the key words of a model that is based on the parton model rather than the string model for low  $p_T$  processes. In this paper we combine the two, using ECOMB to generate events from which we calculate the erraticity measures. The result should be of considerable interest, since, on the one hand, the erraticity analysis of the NA22 data [5] is currently being carried out, and, on the other, it can motivate the investigation and comparison of erraticities in various different collision processes, ranging from  $e^+e^-$  annihilation to heavy-ion collisions.

The study of erraticity originated in an attempt to understand possible chaotic behaviors in quark and gluon jets [1], since QCD is intrinsically nonlinear. In the search for a measure of chaos it was realized that the fluctuation of the hadronic final states of a parton jet is the only observable feature of the QCD process that can replace the unpredictable trajectories in classical nonlinear dynamics. A multiparticle final state in momentum space is a spatial

pattern. Once a measure is found to quantify the fluctuation of spatial patterns, the usefulness of the method goes far beyond the original purpose of characterizing chaoticity in perturbative QCD processes. Many problems involve spatial patterns; they can range from phase transition in condensed matter to galactic clustering in astrophysics. Even continuous time series can be transformed by discrete mapping to spatial patterns [6]. Thus the erraticity analysis, which is the study of the fluctuation of spatial patterns, is more general than the determination of chaotic behavior. Indeed, we have applied it to the study of phase transition in magnetic systems by use of the Ising model [6], as well as to the characterization of heartbeat irregularities in ECG time series [7].

Multiparticle production at low  $p_T$  has always eluded first-principle calculation because of its nonperturbative nature. Various models that simulate the process can generate the average quantities, but fail in getting correctly the fluctuations from the averages [2]. In particular, few models can fit the intermittency data [5]. To our knowledge ECOMB is the only one that can reproduce those data [4], (apart from its predecessor ECCO [8]). Since that model is tuned to fit the data by the adjustment of several parameter, it is necessary to test its predictions on some new features of the production process. Erraticity is such a feature. The fluctuation of final-state patterns presents a severe test of any model.

ECOMB includes many sources of fluctuations in hadronic collisions. In the framework of the eikonal formalism it allows for fluctuations in impact parameter  $b$ . For any  $b$  there is the fluctuation of the number  $\mu$  of cut Pomerons. For any  $\mu$  there is the fluctuation of the number  $\nu$  of partons. For any  $\nu$  the color distribution along the rapidity axis can still fluctuate initially. During the evolution process the local subprocesses of color mutation, spatial contraction and expansion, branching into neutral subclusters, and hadronization into particles or resonances can all fluctuate. Taken together the model can generate such widely fluctuating events that fitting some average quantity such as  $\langle n \rangle$  or  $dn/dy$  does not explore the full extent of its characteristics. The dependence of normalized factorial moments  $F_q$  on the bin size  $\delta$  usually called intermittency, probes deeper, but it is nevertheless a measure that is averaged over all events. Erraticity is a true measure of event-to-event fluctuation.

## 2 Erraticity

There are various ways to characterize a spatial pattern. We shall use the horizontal factorial moments. Given the rapidity distribution of a particular event, we first convert it to a distribution in the cumulative variable  $X$  [9, 1], in terms of which the average rapidity distribution  $dn/dX$  is uniform in  $X$ . We then calculate from that distribution for that event the normalized  $F_q$

$$F_q = \langle n(n-1) \cdots (n-q+1) \rangle / \langle n \rangle^q \quad , \quad (1)$$

where  $\langle \cdots \rangle$  signifies (horizontal) average over all bins, and  $n$  is the multiplicity in a bin. We emphasize that (1) does not involve any average over events.  $F_q$  does not fully describe the structure of an event, since at any fixed  $q$  it is insensitive to the rearrangement of the bins. However, it does capture some aspect of the fluctuations from bin to bin, and is adequate for our purpose.

Since  $F_q$  fluctuates from event to event, one obtains a (vertical) distribution  $P(F_q)$  after many events. Let the vertical average of  $F_q$  determined from  $P(F_q)$  be denoted by  $\langle F_q \rangle_v$ . Then in terms of the normalized moments for separate events

$$\Phi_q = F_q / \langle F_q \rangle_v \quad , \quad (2)$$

we can define the vertical  $p$ th order moments of the normalized  $q$ th order factorial (horizontal) moments

$$C_{p,q} = \langle \Phi_q^p \rangle_v \quad . \quad (3)$$

Erraticity refers to the power law behavior of  $C_{p,q}$  [1, 3]

$$C_{p,q} \propto M^{\psi_q(p)} \quad , \quad (4)$$

where  $M$  is the number of bins,  $1/\delta$ , and the length in  $X$  space is 1.  $\psi_q(p)$  is referred to as the erraticity exponent. If the spatial pattern never changes from event to event,  $P(F_q)$  would be a delta function at  $\Phi_q = 1$ , and  $C_{p,q}$  would be 1 at all  $M$ ,  $p$ , and  $q$ , resulting in  $\psi_q(p) = 0$ . The larger  $\psi_q(p)$  is, the more erratic is the fluctuation of the spatial patterns.

Since  $\psi_q(p)$  is an increasing function of  $p$  with increasing slope, an efficient way to characterize erraticity with one number (for every  $q$ ) is simply to use the slope at  $p = 1$ , i.e.

$$\mu_q = \left. \frac{d}{dp} \psi_q(p) \right|_{p=1} \quad . \quad (5)$$

It is referred to as the entropy index [1]. Experimentally, it is easier to determine first an entropy-like quantity  $\Sigma_q$  directly from  $\Phi_q$ :

$$\Sigma_q = \langle \Phi_q \ln \Phi_q \rangle_v \quad , \quad (6)$$

which follows from (3) and

$$\Sigma_q = dC_{p,q}/dp|_{p=1} \quad , \quad (7)$$

and then to determine  $\mu_q$  from  $\Sigma_q$  using

$$\mu_q = \frac{\partial \Sigma_q}{\partial \ln M} \quad , \quad (8)$$

provided that  $C_{p,q}$  has the scaling behavior (4). In [1] it is found that  $\mu_q$  is larger for quark jets than for gluon jets, indicating that the branching process of the former is more chaotic, or, in more words, the event-to-event fluctuation is more erratic.

If the moments  $C_{p,q}$  do not have the exact scaling behavior in  $M$ , as in (4), but have similar nonlinear dependences on  $M$ , we can consider a generalized form of scaling

$$C_{p,q}(M) \propto g(M)^{\tilde{\psi}(p,q)} \quad . \quad (9)$$

If (9) is approximately valid for a common  $g(M)$  for all  $p$  and  $q$ , it then follows from (7) that

$$\Sigma_q(M) \propto \tilde{\mu}_q \ln g(M) \quad , \quad (10)$$

where

$$\tilde{\mu}_q = \left. \frac{d}{dp} \tilde{\psi}(p, q) \right|_{p=1} . \quad (11)$$

Despite the similarity between (5) and (11),  $\tilde{\mu}_q$  is distinctly different from  $\mu_q$  and should not be compared to one another unless  $g(M) = M$ .

If (10) is indeed good for a range of  $q$  values, then we expect a linear dependence of  $\Sigma_q$  on  $\Sigma_2$  as  $M$  is varied. Let the slope of such a dependence be denoted by  $\omega_q$ , i.e.,

$$\omega_q = \frac{\partial \Sigma_q}{\partial \Sigma_2} . \quad (12)$$

Then we have

$$\tilde{\mu}_q = \tilde{\mu}_2 \omega_q . \quad (13)$$

A variation of this scheme that makes use of an extra control parameter  $r$  in the problem is considered in [6]. It is found there that the entropy indices determined that way are as effective as Lyapunov exponents in characterizing classical nonlinear dynamical systems.

### 3 Scaling Behaviors

The erraticity analysis described above involves only measurable quantities, so it can be directly applied to the experimental data. The NA22 data at  $\sqrt{s} = 22$  GeV are ideally suited for this type of analysis, since  $F_q$  fluctuates widely from event to event [5]. The nuclear collision data, such as those of NA49, can also be studied, but  $p_T$  cuts should be made to reduce the hadron multiplicity to be analyzed, thereby enhancing the erraticity to be quantified.

Here we apply the analysis to hadronic collisions generated by ECOMB. The parameters are tuned to fit  $\langle n \rangle$ ,  $P_n$ ,  $dn/dy$  and  $\langle F_q \rangle_v$  of the NA22 data [5]. Without any further adjustment of the parameters in the model we calculate  $C_{p,q}(M)$ , which are therefore our predictions for hadronic collisions at 22 GeV. The results from simulating  $3 \times 10^4$  Monte Carlo events are shown on the left side of Fig. 1. The lines are drawn to guide the eye.

From the points shown, it is clear that the dependences of  $C_{p,q}$  on  $M$  in the log-log plots are not very linear, especially for the more reliable cases of  $q = 2$  and 3, where the statistics are higher. Thus the power-law behavior in (4) is not well satisfied. Since the general behaviors of  $C_{p,q}$  are rather similar in shape, we can regard  $C_{2,2}$  as the reference that carries the typical dependence on  $M$ , and examine  $C_{p,q}$  vs  $C_{2,2}$  when  $M$  is varied as an implicit variable. The results are shown on the right side of Fig. 1. We have left out the highest points that correspond to the smallest bin size, since they show saturation at  $q > 2$ . We have also left out the points corresponding to  $\ln M = 0$ , since the scaling behaviors do not extend to the biggest bin size. The straightlines are linear fits of the points shown and lend support to the scaling behavior

$$C_{p,q} \propto C_{2,2}^{\chi(p,q)} . \quad (14)$$

The slopes of the fits are  $\chi(p, q)$ , which are shown in Fig. 2. One may regard  $\chi(p, q)$  as a representation of the erraticity properties of the particle production data, when there is no strict scaling law as in (4).

The behavior of  $\chi(p, q)$  exhibited in Fig. 2 can be described analytically, if we fit the points by a quadratic formula for each  $q$ . The result is shown by the lines in Fig. 2. Evidently, the fits are excellent. The properties of the smooth behaviors can be further summarized by their derivatives at  $p = 1$ :

$$\chi'_q \equiv \left. \frac{d}{dp} \chi(p, q) \right|_{p=1} . \quad (15)$$

The values of  $\chi'_q$  are 0.834, 2.818, 5.243 and 7.847 for  $q = 2, \dots, 5$ , and are shown in Fig. 3. We suggest that these values of  $\chi'_q$  be used to compare with the experimental data.

Although  $C_{p,q}(M)$  do not satisfy (4), we can consider the more general form (9). If the same function  $g(M)$  is good enough in (9) for all  $p$  and  $q$ , then it follows from (14) that

$$\chi(p, q) = \tilde{\psi}(p, q) / \tilde{\psi}(2, 2) . \quad (16)$$

Using (11) we then have

$$\tilde{\mu}_q = \tilde{\psi}(2, 2) \chi'_q . \quad (17)$$

It should be noted that, whereas  $\chi'_q$  follows only from the scaling property of (14), the determination of  $\tilde{\psi}(2, 2)$ , and therefore  $\tilde{\mu}_q$ , requires the knowledge of  $g(M)$  in (9).

To determine  $g(M)$ , we write it in the form

$$\ln g(M) = (\ln M)^a . \quad (18)$$

By varying  $a$ , we can find a good linear behavior of  $\ln C_{2,2}$  vs  $\ln g(M)$ , as shown by the dashed line in Fig. 4 for  $a = 1.8$ . The corresponding value of  $\tilde{\psi}(2, 2)$  determined by the slope of the straightline fit is 0.119. Using that in (17) yields a set of values of  $\tilde{\mu}_q$ , which are shown in Fig. 5 by the open-circle points. In particular, we have

$$\tilde{\mu}_2 = 0.099 , \quad (19)$$

a quantity that has a separate significance below.

We remark that in checking the validity of (9) for values of  $p$  and  $q$  other than 2, one can improve the linearity of the points for each  $p$  and  $q$  by slight adjustments of the value of  $a$ . If there is a range of possible  $g(M)$  that depends on  $p$  and  $q$  to yield the best fits, however small the variations in  $a$  may be, the scheme defeats the point of defining a universal  $\tilde{\psi}(p, q)$ . We thus propose that the emphasis of the erraticity analysis should be placed on (14), which is independent of  $g(M)$ , and that (9) is examined only for  $p = 2, q = 2$  so that (17) can be evaluated.

Since  $\tilde{\mu}_q$  is distinct from  $\mu_q$ , we cannot compare our result on  $\tilde{\mu}_q$  with the theoretical values of  $\mu_q$  found for quark and gluon jets [1], nor with the experimental values of  $\mu_q$  determined from  $pp$  collisions at 400 GeV/c (NA27) [10].

The values of  $\tilde{\mu}_q$  can also be determined independently by use of  $\Sigma_q(M)$ . From the definition in (6) we have calculated  $\Sigma_q$  as functions of  $\ln M$ , as shown in Fig. 6(a). Not surprisingly, the dependences are not linear. However, when  $\Sigma_q$  is plotted against  $\Sigma_2$  in Fig. 6(b), they all fall into straightlines, except for the point corresponding to the smallest bin for  $q = 5$  (which we have left out for the fit). The slopes, which give  $\omega_q$  defined in (12), are 1.0, 3.244, 6.0, and 9.101 for  $q = 2, \dots, 5$ . They are shown in Fig. 7. If we examine (10) for  $q = 2$  only, and plot  $\Sigma_2$  vs  $\ln g(M)$  with  $a = 1.8$ , as in Fig. 4, we obtain a linear behavior with a slope

$$\tilde{\mu}_2 = 0.095 \quad . \quad (20)$$

This value is to be compared with that in (19) with only 4% discrepancy. Of the two methods of determining  $\tilde{\mu}_2$ , this latter approach is more reliable, since the derivative in  $p$  at  $p = 1$  is done analytically in the definition of  $\Sigma_q$  in (7), whereas in the former approach the differentiation is done in (15) using the fitted curve in Fig. 2. Substituting (20) into (13), we can determine the values of  $\tilde{\mu}_q$  for  $q > 2$  from the values of  $\omega_q$  in Fig. 7. The result is shown by the solid points in Fig. 5. Clearly, the two methods yield essentially the same result.

Another way to check the degree of consistency of the two methods, independent of the details on  $g(M)$ , is to examine the ratio  $r_q = \chi'_q/\omega_q$ . The quantities in that ratio are derived from the straightline fits of  $\ln C_{p,q}$  vs  $\ln C_{2,2}$  and  $\Sigma_q$  vs  $\Sigma_2$  (as in Fig. 1 and Fig. 6) without resorting to such equation as (18). According to (13) and (17), the ratio  $r_q$  should be a constant, independent of  $q$ . From the values of  $\chi'_q$  and  $\omega_q$  given above in connection with Figs. 3 and 7, we find that  $r_q = 0.834, 0.867, 0.874$ , and  $0.862$  for  $q = 2, 3, 4, 5$ . The average is 0.86, so the standard deviation is at the 1-2% level. Evidently, the two methods are quite consistent, whatever  $g(M)$  may be. From (13) and (17), one would expect  $r_q$  to be  $\tilde{\mu}_2/\tilde{\psi}(2, 2)$ , which according to the numbers given in Fig. 4, is 0.798. The discrepancy from 0.86 is nearly 7%. Thus the disagreement of the values of  $\tilde{\mu}_q$  in Fig. 5, though not large, has the same root as the disagreement between (19) and (20), namely, the necessity to use a specific form of  $g(M)$ . Nevertheless, at the level of inaccuracy of 4%, which is comparable to the typical uncertainty in the experimental data, the value of  $\tilde{\mu}_2$  given by either (19) or (20) clearly provides an effective measure of erraticity in soft production.

## 4 Conclusion

In conclusion, we recapitulate the two essential points of this paper. One is the prediction of ECOMB on the nature of fluctuations of the factorial moments  $F_q$  from event to event. The other is the proposed method of summarizing the scaling behaviors of  $C_{p,q}$  that do not have strict power-law dependences on the bin size. The two aspects of this paper converge on the new erraticity measures  $\chi(p, q)$ ,  $\chi'_q$ ,  $\omega_q$  and  $\tilde{\mu}_q$ .

It is hoped that the data from both NA22 and NA27 can be analyzed in terms of these measures so that the dynamics of soft interaction contained in ECOMB can be checked by the experiments.

The proposed measures of erraticity are, of course, more general than the application made here to soft production. Event-to-event fluctuation has recently become an important theme in collisions of all varieties:  $e^+e^-$  annihilation, leptonproduction, hadronic collisions at

very high energies where hard subprocesses are important, and heavy-ion collisions. What was lacking previously is an efficient measure of such fluctuations. The erraticity measures proposed in [1, 3], now generalized to  $\chi(p, q)$ ,  $\chi'_q$ ,  $\omega_q$  and  $\tilde{\mu}_q$  are well suited for that purpose. They may be redundant, if strict scaling in  $M$  is good enough to give the erraticity indices  $\psi(p, q)$ . The method of treating less-strict scaling properties proposed here may well be more generally applicable to the wide range of collision processes amenable to erraticity study.

### Acknowledgments

This work was supported in part by U.S. Department of Energy under Grant No. DE-FG03-96ER40972 and by the National Science Foundation under contract No. PHY-93-21949.

### References

- [1] Z. Cao and R. C. Hwa, Phys. Rev. Lett. **75**, 1268 (1995); Phys. Rev. D **53**, 6608 (1996); **54**, 6674 (1996).
- [2] E. A. DeWolf, I. M. Dremin, and W. Kittel, Phys. Rep. **270**, 1 (1996).
- [3] R. C. Hwa, Acta Physica Polonica B**27**, 1789 (1996); and in *Correlations and Fluctuations*, Proceedings of the Nijmegen Workshop on Multiparticle Production, edited by R. C. Hwa, W. Kittel, W. J. Metzger, and J. Schotanus, *Correlations and Fluctuations* (World Scientific, Singapore, 1997).
- [4] Z. Cao and R. C. Hwa, hep-ph/9808399, submitted to Phys. Rev. D.
- [5] I. V. Ajineko *et al.* (NA22), Phys. Lett. B **222**, 306 (1989); **235**, 373 (1990).
- [6] Z. Cao and R. C. Hwa, Phys. Rev. E **56**, 326 (1997).
- [7] R. C. Hwa, physics/9809041, submitted to Phys. Rev. Lett.
- [8] R. C. Hwa and J. C. Pan, Phys. Rev. D **45**, 106 (1992); J. C. Pan and R. C. Hwa, Phys. Rev. D **48**, 168 (1993).
- [9] A. Białas and M. Gardzicki, Phys. Lett. B **252**, 483 (1990)
- [10] W. Wang and Z. Wang, Phys. Rev. D **57**, 3036 (1998).

## Figure Captions

- Fig. 1** Log-log plots of  $C_{p,q}$  versus  $M$  on the left side and versus  $C_{2,2}$  on the right side. The lines on the left side are to guide the eye, while the ones on the right side are linear fits.
- Fig. 2** The slopes of the linear fits on the right side of Fig. 1 are plotted against  $p$  for various values of  $q$ . The lines are fits by quadratic formula.
- Fig. 3** The derivatives of  $\chi(p, q)$  in Fig. 2 at  $p = 1$ .
- Fig. 4** The open circles are for  $C_{2,2}$  and the solid points are for  $\Sigma_2$ . The lines are linear fits, whose slopes are  $\tilde{\psi}(2, 2)$  and  $\tilde{\mu}_2$ , respectively.
- Fig. 5**  $\tilde{\mu}_q$  determined in two different ways: Eq. (17) for the open circles and Eq. (13) for the solid points.
- Fig. 6** (a)  $\Sigma_q$  vs  $\ln M$  for various  $q$ ; (b)  $\Sigma_q$  vs  $\Sigma_2$  with the lines being linear fits.
- Fig. 7** The slopes of the straightlines in Fig. 6(b),  $\omega_q$ , plotted against  $q$ .













